Cumulative Review

Ex: let $g(x) = 2x^2 + x - 1$. Find a value c between 1 and 4 such that the ARoC from $x=1$ to $x=4$ is equal to the IRoC at $x=c$.

$$\frac{g(4)-g(1)}{4-1} = g'(c)$$

$$\frac{(2(4)^2+4-1)-(2(1)^2+1-1)}{3} = (4x+1)\Big|_c$$

$$\frac{35-2}{3} = 4c+1$$

$$11 = 4c+1$$

$$10 = 4c$$

$$\boxed{c = \frac{5}{2}}$$

Ex: Compute the limits.

a) $\lim_{x\to 2} (3x+1)(e^x)$

$= \left(\lim_{x\to 2} 3x+1\right)\left(\lim_{x\to 2} e^x\right)$

$= (3(2)+1)e^2 = \boxed{7e^2}$

b) $\lim_{x\to 1} \frac{6}{(x-1)^3}$

"$\frac{6}{0}$" $= \boxed{\text{DNE}}$

c) $\lim_{x\to 0} \frac{x^3+5x^2+6x}{x^2+2x}$

"$\frac{0}{0}$" $= \lim_{x\to 0} \frac{\cancel{x}(x^2+5x+6)}{\cancel{x}(x+2)}$

$= \lim_{x\to 0} \frac{x^2+5x+6}{x+2} = \frac{6}{2} = \boxed{3}$

d) $\lim_{x\to\infty} \dfrac{3x^3+6}{5x^3+x^2+1}$

$= \lim_{x\to\infty} \dfrac{3\cancel{x^3}}{5\cancel{x^3}}$

$= \lim_{x\to\infty} \dfrac{3}{5} = \boxed{\dfrac{3}{5}}$

Ex: let $f(x) = \dfrac{3}{x}$. Write the equation of the tangent line to the graph of f at $x = -3$.

Point $f(-3) = \dfrac{3}{-3} = -1$ $(-3, -1)$

Slope $f'(-3)$

$f(x) = 3x^{-1}$ $f'(x) = -3x^{-2} = \dfrac{-3}{x^2}$

$f'(-3) = \dfrac{-3}{(-3)^2} = \dfrac{-3}{9} = \dfrac{-1}{3}$ $m = \dfrac{-1}{3}$

Point slope form

$y-(-1) = \dfrac{-1}{3}(x-(-3))$

$y+1 = \dfrac{-1}{3}(x+3)$

$\boxed{y = \dfrac{-1}{3}x - 2}$

Ex: Find the derivatives.

a) $f(x) = 6 \ln(x+2)$

$$f'(x) = 6 \cdot \frac{1}{x+2} (1)$$

$$= \boxed{\frac{6}{x+2}}$$

b) $g(x) = (e^{x+1})(3x+5)^2$

*product rule

$$g'(x) = (e^{x+1})(1)(3x+5)^2 + (e^{x+1})(2)(3x+5)(3)$$

$$= \boxed{(3x+5)^2 e^{x+1} + 6(3x+5)e^{x+1}}$$

c) $h(x) = \dfrac{4x^2+1}{\ln(3x)}$ * quotient rule

$$\boxed{h'(x) = \frac{(8x)\ln(3x) - (4x^2+1)\cdot\frac{1}{3x}(3)}{[\ln(3x)]^2}}$$

<u>Ex</u>: let $f(x) = \sqrt[3]{x^2}$. find $f''(x)$.

$f(x) = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3}$

$f''(x) = \frac{2}{3}\left(\frac{-1}{3}\right)x^{-4/3}$

$= \frac{-2}{9}x^{-4/3} = \boxed{\frac{-2}{9x^{4/3}}}$

Ex: The half-life of an isotope is 2 years. Suppose we have a 40 gram sample. How much of the sample will remain after 5 years?

$$P = P_0 e^{rt}$$

$$20 = 40e^{r(2)}$$

$$\frac{1}{2} = e^{2r}$$

$$\ln\frac{1}{2} = \cancel{\ln e}^{2r}$$

$$\ln\frac{1}{2} = 2r$$

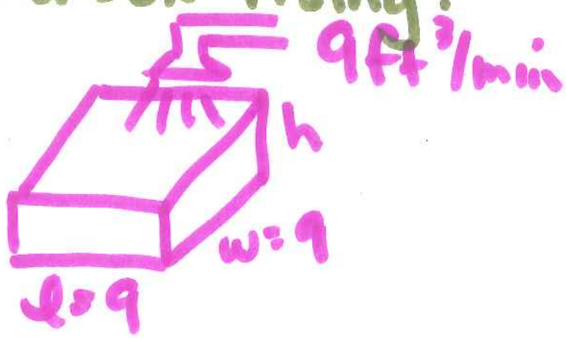
$$r = \frac{\ln\frac{1}{2}}{2}$$

$$P = P_0 e^{rt}$$

$$= 40e^{\frac{\ln\frac{1}{2}}{2}(5)}$$

$$= \boxed{7.07106 \text{ grams.}}$$

Ex: a sandbox with square base is being filled with sand at a rate of 9 ft^3/min. The sandbox is 9ft long and 9 ft wide. How fast is the level of the sand in the sandbox rising?



$$V = l \cdot w \cdot h$$
$$= 9 \cdot 9 \cdot h$$
$$= 81h$$

differentiate $V = 81h$ with respect to t!

$$\frac{dV}{dt} = 81 \cdot \frac{dh}{dt}$$

$$9 = 81 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{9}{81} = \boxed{\frac{1}{9} \text{ ft/min}}$$

Ex: Find the max and min values for $f(x) = x^3 - 3x^2 - 9x + 5$ on $[0,4]$.

$$f'(x) = 3x^2 - 6x - 9$$
$$= 3(x^2 - 2x - 3)$$
$$= 3(x-3)(x+1)$$

$f'(x) = 0$ when $x = 3, \cancel{-1}$ -1 is not in interval $[0,4]$.

f and f' are polynomials - defined, cont, diff everywhere ✓

check:

$f(0) = 0^3 - 3(0)^2 - 9(0) + 5 = 5$ ← max

$f(3) = 3^3 - 3(3)^2 - 9(3) + 5 = -22$ ← min

$f(4) = 4^3 - 3(4)^2 - 9(4) + 5 = -15$

$\boxed{\begin{array}{l}\text{min value } -22 \text{ occurs at } x=3 \\ \text{max value } 5 \text{ occurs at } x=0.\end{array}}$